ÉRETTSÉGI VIZSGA • 2021. május 4.

MATEMATIKA ANGOL NYELVEN

EMELT SZINTŰ ÍRÁSBELI VIZSGA

2021. május 4. 9:00

Időtartam: 300 perc

Pótlapok száma	
Tisztázati	
Piszkozati	

EMBERI ERŐFORRÁSOK MINISZTÉRIUMA

Instructions to candidates

1. The time allowed for this examination paper is 300 minutes. When that time is up, you will have to stop working.

2. You may solve the problems in any order.

3. In part II, you are only required to solve four of the five problems. **When you have finished the examination, enter the number of the problem not selected in the square below**. *If it is not clear* for the examiner which problem you do not want to be assessed, the last problem in this examination paper will not be assessed.

4. On solving the problems, you may use a calculator that cannot store and display textual information. You may also use any edition of the four-digit data tables. The use of any other electronic device or printed or written material is forbidden!

5. **Always write down the reasoning used to obtain the answers. A major part of the score will be awarded for this.**

6. **Make sure that calculations of intermediate results are also possible to follow.**

7. **The use of calculators** in the reasoning behind a particular solution **may be accepted without further mathematical explanation in case of the following operations**: addition, subtraction, multiplication, division, calculating powers and roots, $n!$, $\binom{n}{k}$, replacing the tables found in the 4-digit Data Booklet (sin, cos, tan, log, and their inverse functions), approximate values of the numbers π and e, finding the solutions of the standard quadratic equation. No further explanation is needed when the calculator is used to find the mean and the standard deviation, as long as the text of the question does not explicitly require the candidate to show detailed work. **In any other cases, results obtained through the use of a calculator are considered as unexplained and points for such results will not be awarded.**

8. On solving the problems, theorems studied and given a name in class (e.g. the Pythagorean Theorem or the height theorem) do not need to be stated precisely. It is enough to refer to them by name, but their applicability needs to be briefly explained. Reference to other theorems will be fully accepted only if the theorem and all its conditions are stated correctly (proof is not required) and the applicability of the theorem to the given problem is explained.

9. Always state the final result (the answer to the question of the problem) in words, too!

10. Write in pen. Diagrams may be drawn in pencil. The examiner is instructed not to mark anything written in pencil, other than diagrams. If you cancel any solution or part of a solution by crossing it over, it will not be assessed.

11. Only one solution to each problem will be assessed. In case of more than one attempt to solve a problem, **indicate clearly** which attempt you wish to be marked.

12. Please, **do not write in the grey rectangles**.

I.

1. **a)** Prove that the product of any six consecutive natural numbers is divisible by 45.

b) Is it true that the product of any five consecutive **odd** natural numbers is divisible by 45? (Explain your answer.)

c) How many different solutions does the equation $45 = 3 + 5 + a + b + c$ have, where a, b and c are different **odd** natural numbers and $5 < a < b < c$ is also true?

d) Determine the truth value (true or false) of the statement $(A \vee B) \rightarrow C$ for the various possible truth values of statements A, B, and C and fill in the truth table accordingly. (In **this** part it is not necessary to explain your answers.)

A	B	C	$(A \vee B) \rightarrow C$
t	*t*	*t*	
t	*t*	*f*	
t	*f*	*t*	
t	*f*	*f*	
f	*t*	*t*	
f	*t*	*f*	
f	*f*	*t*	
f	*f*	*f*	

a)	3 points	
b)	3 points	
c)	4 points	
d)	3 points	
T:	13 points	

2. The Budapest Castle Hill Funicular was built in 1870. According to contemporary sources, the angle of inclination of the funicular (hillside rail line) was 33° on the original plans but this, for reasons since forgotten, had been changed to 30° by Henrik Wohlfarth, general contractor of the construction. The location of the upper terminus remained unchanged during construction works, but the lower terminus had been built 6 metres higher than planned. (The imaginary line connecting the planned and actual locations of the lower terminus is perpendicular to the ground surface.)

a) Calculate the length and elevation (height) of the track.

According to the records, the funicular carried a total of 670 thousand passengers in the millennial year of 1896. Assume the funicular was running for 14 hours a day and that it was closed for a day every second week for repairs. This gives about 340 days of operation during the year. The average time interval between rides was 10 minutes. At that time, each car of the funicular had a maximum capacity of 22 passengers. The funicular service consists of two cars. During each ride one of the cars is travelling uphill while the other down, simultaneously.

b) Based on the data above calculate the average percentage of the occupancy rate of the seats in the year 1896.

a)	7 points	
b)	5 points	
T:	12 points	

3. There are 32 students attending a university lecture. Should 4 of the girls leave, more than 60% of the remaining students would be boys. On the other hand, should 6 more girls join the 32 students already in the lecture hall, more than half of all those present would be girls.

a) How many boys and how many girls are attending the lecture?

60% of the many thousand students attending the university are boys, 40% are girls. (This may also be interpreted as follows: the probability that a randomly selected student at this university will be a boy is 0.6, while the probability that such student will be a girl is 0.4.)

b) Suppose that 4 random student sit together at a table in the cafeteria of the university. What is the probability that there will be more boys among them than girls?

When 3 female university students randomly meet, the probability that all three of them are active sportswomen is 0.008.

c) What proportion of the girls are active sportswomen?

a)	6 points	
b)	4 points	
c)	3 points	
T:	13 points	

4. Given are the parabola $x^2 - 4y = 0$ and the line g: $x - y = 5$.

a) Prove that the focus of the parabola is $F(0; 1)$.

b) Give the equation of the circle that has its centre on the line g and crosses through the point $P(0; -1)$ as well as the focus F of the parabola.

c) Give the equation of the tangent drawn to the parabola that is parallel to the line g.

a)	3 points	
b)	5 points	
c)	5 points	
T:	13 points	

II.

You are required to solve any four out of the problems 5 to 9. Write the number of the problem NOT selected in the blank square on page 2.

5. Mr. Kovács has bought a new car for 5 million forints. A mathematical model predicts that the car will lose 12% of its current value each year.

a) Considering this depreciation only, how many full years will pass until the value of Mr. Kovács's car drops below 1.5 million forints?

Precíz Ltd. calculates the current value of cars using monthly depreciation. (Monthly depreciation means that the value of the car decreases by the same percentage each month as compared to the value in the previous month.)

b) Given that the annual depreciation is 12%, show that the monthly depreciation is about 1.06%.

Mr. Kovács's car is in good general condition. He would like to sell it to Precíz Ltd.
A brochure of Precíz Ltd. says that the company uses two different methods to calculate the current value of used cars and the one that is more beneficial to the seller will be applied.
Method 1: the actual age of the car is reduced by 12 months and then a monthly 1.06% depreciation rate is applied for the remaining months.
Method 2: the "age" of the car is estimated by the number of kilometres run, assuming an average of 15 000 kilometres each year. However, with this method the monthly depreciation rate is 1.2% (instead of 1.06%) and the 12-month deduction from the age is not applied either.

c) Which method is more beneficial to Mr. Kovács, assuming that the age of his car is 8 years and 5 months, and it has run a total of 91 250 kilometres so far?

a)	5 points	
b)	4 points	
c)	7 points	
T:	16 points	

You are required to solve any four out of the problems 5 to 9. Write the number of the problem NOT selected in the blank square on page 2.

6. **a)** Throw a fair dice six times. The single mode, the median, and the mean of the numbers thrown, in this particular order, form three consecutive terms of a (strictly monotone) increasing arithmetic progression.
Give an example of such a sequence of numbers thrown with the dice and prove that your example satisfies the conditions above.
Is it true that the standard deviation of the six numbers thrown is also a term of the same arithmetic progression?

b) Throw a fair dice three times. Calculate the probability that the second number thrown will be the mean of the other two numbers.

a)	8 points	
b)	8 points	
T:	16 points	

You are required to solve any four out of the problems 5 to 9. Write the number of the problem NOT selected in the blank square on page 2.

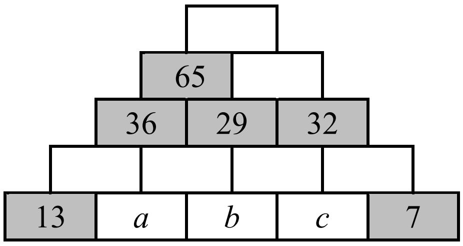
7. **a)** A cylinder is made of solid wood. The height of the cylinder is 30 cm, the total surface area is 10 000 cm^2. The cylinder is transformed into a circular cone whose base circle and height are both identical to those of the original cylinder.
What percentage of the volume of the cylinder will turn to wood shavings and what is the volume of the cone?

b) Consider all possible cylinders with a surface area of 10 000 cm^2. Determine the base radius and height of the one that has the largest volume.

a)	7 points	
b)	9 points	
T:	16 points	

You are required to solve any four out of the problems 5 to 9. Write the number of the problem NOT selected in the blank square on page 2.

8. Number pyramid is a popular puzzle game. The player has to fill the blank cells of the pyramid with positive integers, so that the sum of the numbers in two adjoining cells is always equal to the number in the cell above them.
The numbers 13, 7, 36, 29, 32, and 65 are given in the number pyramid shown.



a) Calculate the values of a, b, and c.

In 1852, while colouring a map of England's counties, a London student found that "correct" colouring requires no more than four colours. (A colouring is considered correct if counties with a common borderline are of different colours.) The generalisation of the student's conjecture for any map (Four colour theorem) was an unsolved mathematical problem for a long time.
The section of the map shows Tolna and the four adjoining counties. The five counties will be coloured in no more than four colours (red, yellow, blue and green).

b) How many different colourings are possible?
(Two colourings are different if there is at least one county in them that is coloured differently.)

a)	9 points	
b)	7 points	
T:	16 points	

You are required to solve any four out of the problems 5 to 9. Write the number of the problem NOT selected in the blank square on page 2.

9. **a)** Prove that $\frac{2}{(n+1)^2-1}=\frac{1}{n}-\frac{1}{n+2}$ ($n \in \mathbf{N}^+$).

b) Calculate the sum of the first four terms of the sequence $a_n=\frac{2}{(n+1)^2-1}$.

Give your answer in $\frac{a}{b}$ form where a and b are positive integer co-primes (relative primes).

c) Calculate the limit $\lim_{n\to\infty}(a_1+a_2+...+a_n)$.

a)	3 points	
b)	3 points	
c)	10 points	
T:	16 points	

	number of question	score			
		maximum	awarded	maximum	awarded
Part I	1	13		**51**	
	2	12			
	3	13			
	4	13			
Part II		16		**64**	
		16			
		16			
		16			
		← question not selected			
Total score on written examination				**115**	

date

examiner

	pontszáma **egész számra** kerekítve	
	elért	programba beírt
I. rész		
II. rész		

dátum

dátum

javító tanár

jegyző